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NUMERICAL SIMULATION OF A DEVELOPED SHEAR TURBULENCE

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Results of two-dimensional numerical studies of turbulence that arises at the interface of two flows of poorly compressible gases are described. The results were obtained using a MAKh software system. The interrelation between spatial and time problems on the development of a turbulent zone induced by shear instability is analyzed. A constant that characterizes the degree of turbulent shear mixing is calculated. The effect of the density difference of the mixing fluids on the growth rate of the turbulence zone is studied. For all density differences considered, the coefficient of heterogeneity of the resultant mixture is evaluated.

Problem Statement. When two incompressible semi-infinite streams of a fluid (or a gas) move in one direction with different velocities U_1 and U_2, turbulence develops at the interface between them. This motion is self-similar; therefore, the thickness of the turbulent mixing zone can be represented as

$$L = 0.5\alpha_u f(\rho_1/\rho_2)|U_1 - U_2|t, \tag{1}$$

where α_u is a constant determined experimentally, t is the time of interaction of the two flows, and $f(\rho_1/\rho_2)$ is a dimensionless function that depends on the density difference and normalized so that $f(1) = 1$. The difference $|U_1 - U_2|$ is an invariant of the Galilean transform, and the flow pattern remains unchanged on interchanging the two flow velocities: $U_1 \leftrightarrow U_2$. This interchanging is equivalent to the case where the densities are interchanged: $\rho_1 \leftrightarrow \rho_2$. Hence, the function $f(x)$ possesses the following property:

$$f(x) = f(1/x). \tag{2}$$

Since any experimental study in such a time statement of the problem of interest seems to be difficult to perform, the shear instability is usually studied within the following spatial formulation. Two incompressible fluids (or two gases) with densities ρ_1 and ρ_2 are considered. These fluids move along the semiplane $y = 0$, $x < 0$ that separates them, with velocities U_1 and U_2, respectively ($U_1 > U_2$) (Fig.1). We define the Atwood number as $A = (\rho_1 - \rho_2)/(\rho_1 + \rho_2)$. In the case under consideration, this number can be either positive or negative. At the point $x = 0$, the fluids are in contact, while at $x > 0$, turbulence develops at the interface (region I in Fig. 1).

The spatial problem can be substituted with a time one using the relation $x = U_0 t$. The physical meaning of the velocity U_0 becomes clear from the following consideration. If, at $t = 0$, the observer is located at the point $x = 0$, $y = 0$, and, afterwards, he moves along the x-axis with velocity U_0, then, for him, the development of the mixing zone is described by relation (1). Since, at a time $t > 0$, the observer is situated at the point $x = U_0 t$, the width of the mixing zone can be represented as

$$L = 0.5\alpha_u f(\rho_1/\rho_2)|U_1 - U_2|(x/U_0). \tag{3}$$

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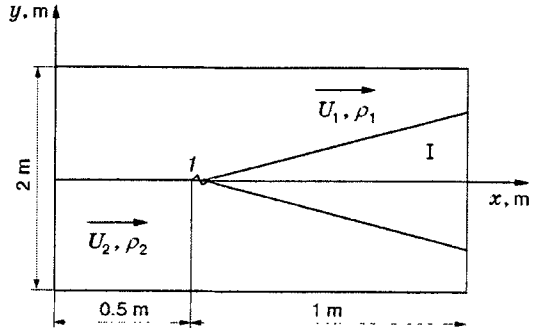

Fig. 1. Scheme of the flow of two fluid streams moving with different velocities.

Some methods for determining the effective velocity U_0 were proposed by Brown and Roshko [1] who considered the following expressions:

$$U_0 = \frac{1}{2}(U_1 + U_2), \qquad U_0 = \frac{1}{2}(U_1 + U_2)\sqrt{1 + \frac{1}{3}\left(\frac{U_1 - U_2}{U_1 + U_2}\right)^2},$$
$$U_0 = \frac{1}{2}(U_1 + U_2)\left(1 + \frac{8}{15\pi}\left(\frac{U_1 - U_2}{U_1 + U_2}\right)^2\right). \tag{4}$$

The velocity U_0 was obtained as the velocity of the streamline with the density $\rho = (\rho_1 + \rho_2)/2$.

Yakovlevskii [2] proposed the relation

$$U_0 = (U_1\rho_1 + U_2\rho_2)/(\rho_1 + \rho_2). \tag{5}$$

Abramovich [3] indicated that, for a large difference in density of the mixing substances, the effective velocity is given by the integral relation

$$U_0 = \int_{y_1}^{y_2} \rho U\, dy \Big/ \int_{y_1}^{y_2} \rho\, dy, \tag{6}$$

where y_1 and y_2 are the boundaries of the mixing zone.

Let us study now relations (4)–(6) for the effective velocity U_0 by comparing the results of two experiments similar to that shown in Fig. 1, where the Atwood number acquires values with the opposite signs. To do this, we may interchange the values of two densities, $\rho_1 \leftrightarrow \rho_2$, being assumed different ($\rho_1 \neq \rho_2$). In this case, property (2) is valid, and hence, the following relation holds:

$$\left(\frac{dL}{dx}\right)_1 \Big/ \left(\frac{dL}{dx}\right)_2 = \frac{U_{02}}{U_{01}}. \tag{7}$$

If, from these experiments, an expression for U_0 is found, then, to determine α_u, it will suffice to perform a third experiment for $\rho_1 = \rho_2$, from which we obtain $\alpha_u = 2(dL/dx)_3 U_{03}/|U_1 - U_2|$. Having the expression for U_0 and the value of α_u at our disposal, from the results of the above experiments and additional tests with different ρ_1/ρ_2 ratios, we can determine the function $f(\rho_1/\rho_2)$.

Thus, the following basic characteristics of turbulent shear mixing can be obtained:

— the characteristic velocity U_0 that interrelates the spatial and time problems;

— the constant α_u that characterizes the degree of turbulent mixing;

— the function $f(\rho_1/\rho_2)$ that determines the relation between the width of the turbulent shear mixing zone and the density difference.

Numerical simulation was performed using a MAKh software system [4], which permits modeling of vortex flows with considerable deformations of the interfaces between the substances.

TABLE 1

Numerical experiment	ρ_1, kg/m^3	ρ_2, kg/m^3	U_1, m/sec	U_2, m/sec	A
1	1.1	7.7	1.30	9.15	0.75
2	1.1	7.7	9.15	1.30	−0.75
3	1.1	1.1	1.30	9.15	0

Initial Data. Numerical Results. A planar flow of two poorly compressible gases was set in a rectangular computational domain divided by the interface $y = 0$ into two strata (see Fig. 1). The parameters of two streams are listed in Table 1. The substances are described by the equation of state for an isothermal gas $P_i = C_0^2(\rho_i - \rho_{0i})$, where P_i is the partial pressure of the ith component of the mixture, $C_0 = 310$ m/sec is the velocity of sound, ρ_i and ρ_{0i} are the densities of the disturbed and undisturbed flows, and the subscript i denotes a mixture component. The computational domain was divided into two strata by the line $y = 0$. The upper and lower boundaries of the system were assumed to be rigid walls, and the left and right edges were stationary Eulerian boundaries. At the Eulerian stage of the calculation procedure, a constant inflow of a substance in each stratum was specified at the left boundary, the substance leaving the computational domain through the right boundary. At the Lagrangian stage, a constant normal velocity was set at the left boundary of each stream, and the normal velocity

$$U_n = c_1 U_1 + c_2 U_2 \tag{8}$$

was specified at the right boundary. Here c_1 and c_2 are the mass concentrations of the first and second substances in interfacial computational cells, and U_1 and U_2 are the velocities of the substances at the inflow boundary. Several types of boundary conditions at the right boundary were considered. All of them, to one extent or another, affect the development of the turbulent mixing zone near the right boundary of the computational domain. Condition (8) was adopted as exerting the smallest effect.

A rigid wall 0.5 m long in the left part of the computational domain and a contact interface over the remaining section were adopted as the boundary conditions at the interface between the strata. The calculation was carried out on a square mesh with a step $\Delta x = \Delta y = 0.01$ m. At the initial time, random disturbances with the highest amplitude of $0.5\Delta y$ were set at the interface between two streams, then the interface was calculated as a Lagrangian line up to the time $t = 0.1$ sec when the width of the disturbed zone increased approximately to $2\Delta y$. From this time on, the calculations were conducted on a stationary Eulerian mesh with allowance for overflow of the substances through the interface between the strata. The development of the mixing zone of two streams was modeled with due regard for their concentrations. To simulate the mixture of two substances, the condition of mechanical equilibrium of the components was used. After the time $t = 0.1$ sec, "vibrator" 1 was installed at a distance of 0.5 m from the left boundary of the computational domain, at the interface between the flows (see Fig. 1). At this point, the vertical velocity was assumed to vary according to the law $V = V_0 \sin(\omega t)$, where $V_0 = 2$ m/sec and $\omega = 400$ rad/sec. The calculations were continued until a developed region of turbulent mixing was formed. The total time covered by the calculations was about 0.8 sec.

Figure 2 shows instantaneous profiles of the volume concentration of the light component of the mixture. Following [1], the self-similar slope $dL/dx = B_0$ was obtained by averaging over four times with the time interval $\Delta t = 0.025$ sec.

In processing the numerical results, for each column of computational cells, we determined the width of the mixing zone using the conditions $f_1(y_{\text{upp}}) = 0.99$ and $f_1(y_{\text{low}}) = 0.01$. Here f_1 is the volume concentration of the first substance and y_{low} and y_{upp} are the lower and upper boundaries of the mixing zone. The resultant dependences $y_{\text{low}}(x_i)$ and $y_{\text{upp}}(x_i)$, where i is the subscript denoting the column number, were approximated by straight lines which were built using the condition of the least standard deviation.

The values of the self-similar slope $dL/dx = B_0$ and its standard deviation ΔB_0 are listed in Table 2. To eliminate the effect caused by non-self-similar sections of the mixing zone located near the "vibrator" and

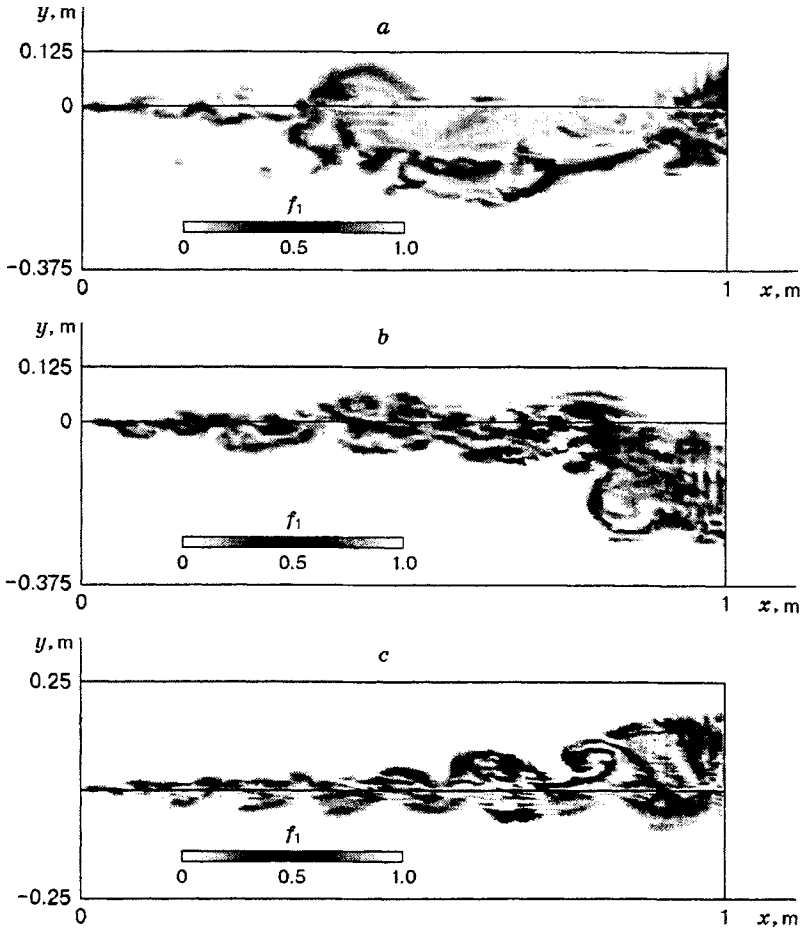

Fig. 2. Distribution of volume concentration: (a) $t = 0.75$ sec and A = −0.75; (b) $t = 0.75$ sec and A = 0; (c) $t = 0.6$ sec and A = 0.75.

TABLE 2

Numerical experiment	B_0	ΔB_0	B_0 [1]	A
1	0.19	0.02	0.22	0.75
2	0.37	0.04	0.41	−0.75
3	0.31	0.02	0.29	0
4	0.21	0.02	0.22	0.75

near the right boundary, it was necessary to reject some computational columns at the boundaries in data processing. The corresponding displacements of the left and right boundaries (Δ_L and Δ_R) were 20 and 10 nodal points, respectively. The values of Δ_L and Δ_R were chosen with allowance for the condition of the least ratio of the standard deviation to the mean value of B_0. To check the effect of initial conditions and mesh steps Δx and Δy on the values obtained for the self-similar slope B_0, we performed another, fourth simulation for A = 0.75 with the smallest width of the turbulent zone; therefore, the initial conditions could exert the greatest effect on the calculation results. The change in the self-similar slope caused by a twofold decrease in both the mesh step and the amplitude of the "vibrator's" oscillations was found to be well within the error of data processing (see Table 2).

According to (7), the ratio of the effective velocities for the case where the Atwood number acquires

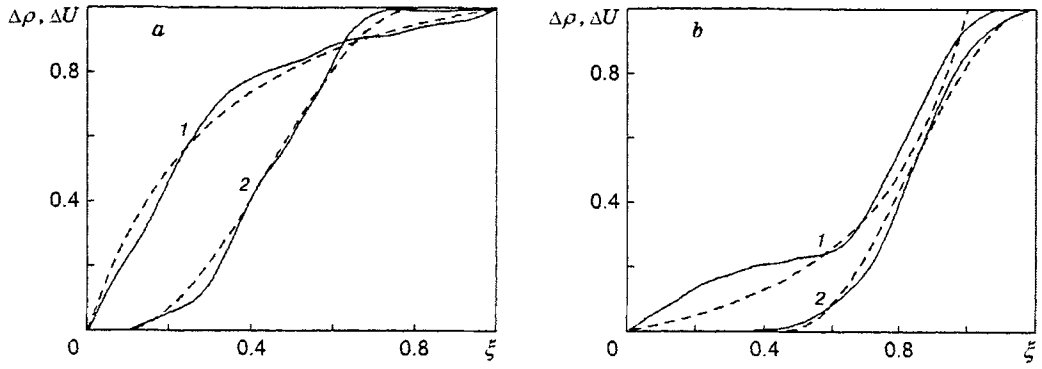

Fig. 3. Profiles of the excess densities ($\Delta\rho$) and velocities (ΔU) for A = −0.75 (a) and 0.75 (b): solid curves are profiles obtained in the present simulations and dashed curves are profiles predicted by the Prandtl theory [3, 5]; curves 1 and 2 refer to density and velocity, respectively.

TABLE 3

A	$\int \Delta U\, d\xi$	$\int \Delta\rho\, d\xi$	$\int \Delta U \Delta\rho\, d\xi$	$U_0/U_{\min}$
−0.75	0.55	0.72	0.49	3.0
0	0.50	—	—	4.0
0.75	0.20	0.34	0.14	5.8

values with the opposite signs is

$$U_{01}/U_{02} = B_{02}/B_{01} = 1.9 \pm 0.2, \qquad B_{01} = 0.20 \pm 0.02.$$

Now we study relations (4)–(6) for the effective velocity U_0 from the viewpoint of their compliance with numerical results. Formulas (4) are independent of the density difference, and hence, we have $U_{01}/U_{02} = 1$. From (5), it follows that $U_{01}/U_{02} = 3.58$. In view of the dependence $U_0 = U_0((\rho_1 + \rho_2)/2)$, we have $U_{01}/U_{02} = 3.7$. From (6), it follows that $U_{01}/U_{02} = 1.9$, which is in good agreement with numerical results.

To calculate the integral velocity and the velocity at the dividing streamline, we used the profiles of the excess velocity $\Delta U(\xi) = (U(\xi) - U_2)/(U_1 - U_2)$ and density $\Delta\rho(\xi) = (\rho(\xi) - \rho(0))/(\rho(1) - \rho(0))$, where $\xi = (y - y_{\text{upp}})/(y_{\text{low}} - y_{\text{upp}})$. The initial velocity and density profiles were recalculated depending on the coordinate ξ for all values of x on the self-similar section, and then these profiles were averaged over the x coordinate. The resultant profiles $\Delta U(\xi)$ and $\Delta\rho(\xi)$ were found to agree well with those predicted by the Prandtl theory [3, 5], which is illustrated by Fig. 3. The theoretical profile $\Delta U(\xi)$ is shown in the ξ' coordinates: $\Delta U(-1) = 0$ and $\Delta U(1) = 1$. In all cases, the turbulent Schmidt number Sc = 0.5 was adopted. The following specific feature of these dependences is observed: the profile $\Delta U(\xi)$ is contracted and shifted relative to the profile $\Delta\rho(\xi)$ with a contraction factor $S \simeq 0.64$–0.70. The latter does not depend on the density difference. This conclusion is in line with a similar dependence $S = (2/3)^{2-2\text{Sc}}$ given by the Prandtl theory.

The effective velocity (6) can be written as

$$U_0 = U_2 + (U_1 - U_2)\Big(\int \Delta U\, d\xi + \Big(\frac{\rho_1}{\rho_2} - 1\Big)\int \Delta U \Delta\rho\, d\xi\Big)\Big/\Big(1 + \Big(\frac{\rho_1}{\rho_2} - 1\Big)\int \Delta\rho\, d\xi\Big).$$

In the limiting cases, as the ratio ρ_1/ρ_2 tends to zero or to infinity, the function $U_0 \to (U_1\rho_1 + U_2\rho_2)/(\rho_1 + \rho_2)$ and takes the values U_2 and U_1, respectively. The calculated ratios $U_0/U_{\min}$ are listed in Table 3. In calculating $U_0/U_{\min}$ for A = 0.75, we took the value at the edge of the turbulent mixing zone equal to $U_2 + 0.9(U_1 - U_2)$ as U_0.

In view of (3), we have $\alpha_u f(\rho_1/\rho_2) = 2B_0U_0/|U_1 - U_2|$. For A = −0.75, 0, and 0.75, we have

$\alpha_u f(\rho_1/\rho_2) = 0.37$, 0.41, and 0.39, i.e., within the accuracy of the calculation error, $\alpha_u f(\mathrm{A}) = \mathrm{const} = \alpha_u$. In other words, in the range of Atwood numbers A = −0.75–0.75, no effect of density difference is observed.

Thus, the results of the present numerical study can be described by the relation $L = (\alpha_u/2)|U_1 - U_2|(x/U_0)$, where $\alpha_u = 0.39 \pm 0.04$ and $U_0 = \int_0^L \rho U\, dy \Big/ \int_0^L \rho\, dy$. The obtained value of the constant α_u that characterizes the degree of turbulent mixing is in agreement with the value $\alpha_u = 0.38$ obtained by Brown and Roshko [1] from the flow patterns determined experimentally.

Heterogeneity Coefficient in the Turbulent Shear Mixing Zone. In [6, 7], the coefficient of heterogeneity of the mixture in the turbulent mixing zone caused by the Rayleigh–Taylor instability, $\theta(\xi) = \langle f_1 f_2\rangle/(\langle f_1\rangle\langle f_2\rangle) = 1 - k_\mathrm{h}$, was estimated, as well as its mean value in the mixing zone

$$\Theta = \int_0^L \langle f_1 f_2\rangle\, dy \Big/ \int_0^L \langle f_1\rangle\langle f_2\rangle\, dy = 1 - \int_0^L k_\mathrm{h}\langle f_1\rangle\langle f_2\rangle\, dy \Big/ \int_0^L \langle f_1\rangle\langle f_2\rangle\, dy.$$

Here f_1 and f_2 are the volume concentrations of the substances, $k_\mathrm{h} = \langle f_1' f_1'\rangle/(\langle f_1\rangle\langle f_2\rangle)$ is the value of the heterogeneity coefficient at a point, the brackets denote averaging in the plane normal to the acceleration vector, and the prime indicates deviation of a quantity from its average value (i.e., fluctuation).

By analogy with [6], we calculated the heterogeneity coefficient in the turbulent shear mixing zone for the profiles of volume concentration. The fluctuations of the volume concentration were averaged in the same manner as was done in finding self-similar profiles of the excess density and velocity. The values of the volume concentration were recalculated depending on the self-similar coordinate $\xi = y/L(x)$ and averaged along the current value of ξ.

The values of the heterogeneity coefficient found for A = −0.75, 0, and 0.75 were $\bar{k}_\mathrm{h} = 0.33$, 0.23, and 0.30, respectively. The value of the heterogeneity coefficient estimated from the experimental data of Bernal and Roshko [8] on the fluctuating volume concentration is $\bar{k}_\mathrm{h} = 0.3$ for A = −0.75. The good agreement between the results of the present two-dimensional numerical studies and those of three-dimensional experiments [8] on determination of the degree of heterogeneity of a turbulent mixture is worth noting.

In [6, 7], it was found that the self-similar heterogeneity coefficient is $k_\mathrm{h} \approx 0.2$ for $\rho_1/\rho_2 = 1$–20 in the case of gravitational mixing. The greater degree of heterogeneity in the case of shear mixing is presumably caused by the more expressed vortex structure formed (see Fig. 2). The origination of a clear vortex structure during shear mixing was also reported in [1, 8].

Conclusions. A series of experiments is proposed that permits determination of the main characteristics of turbulent shear mixing.

Results of numerical simulation are described, in which the divergence of the turbulent zone dL/dx for the velocity and density ratios $U_1/U_2 = 7$ and $\rho_1/\rho_2 = 1/7$, 1, and 7 was observed. The results of two-dimensional numerical studies were found to agree well with experimental data [1, 8]. Self-similar profiles of the excess velocity $\Delta U = (U - U_2)/(U_1 - U_2)$ and density $\Delta\rho = (\rho - \rho_2)/(\rho_1 - \rho_2)$ predicted by the Prandtl theory [3, 5] were obtained.

The expression for the effective velocity $U_0 = \int_L \rho U\, dy \Big/ \int_L \rho\, dy$ proposed by Abramovich [3] is shown to be in agreement with the numerical results obtained.

The constant α_u that characterizes the degree of turbulent mixing is evaluated. Its magnitude was found to equal $\alpha_u = 0.39 \pm 0.04$.

The divergence of the turbulence zone was found to be independent of the density difference in the range of Atwood numbers A = −0.75–0.75. For the above range, we have $f(\rho_1/\rho_2) = 1$.

For all the problems considered, the heterogeneity coefficient of the obtained mixture was calculated. For self-similar problems of shear and gravity turbulence, the mean heterogeneity coefficient over the mixing zone was found to be somewhat higher for the case of shear instability than that for gravity turbulence [6–8].

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